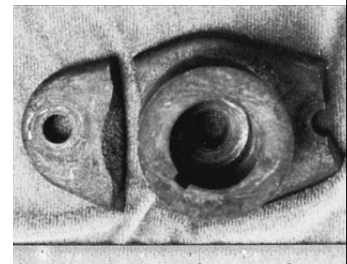


# IPE 303 Product Design I

## Chapter 5: Failures Resulting from Static Loading

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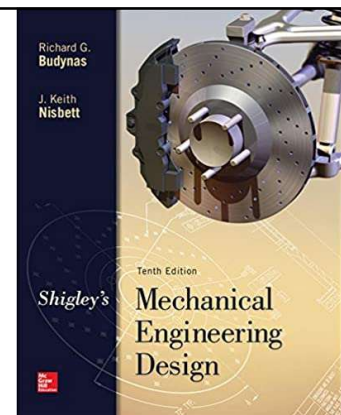


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Reference



Shigley's Mechanical Engineering Design, 10<sup>th</sup> Edition



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## Introduction to Stress

- **Static Load:** A static load is a **stationary force**, or couple applied to a member. To be stationary, the force or couple must be **unchanging in magnitude, point or points of application, and direction**.
- **Stress:** Force per unit area. It is a **vector** quantity.

Difference between **stress and pressure?**

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## Introduction to Stress > Stress Components

### Cartesian Stress Components:

$\sigma_x$  :  $\sigma = \text{Normal Stress}$   
 subscript  $x = \text{direction of the surface normal}$

$\tau_{xy}$  :  $\tau = \text{Shear Stress}$   
 subscript  $x = \text{direction of the surface normal}$   
 subscript  $y = \text{direction of the shear stress}$

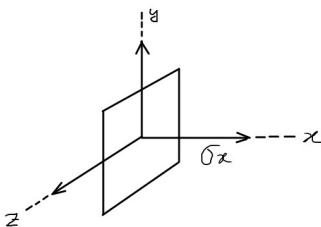


Fig: Stress Components on surface normal to x-axis

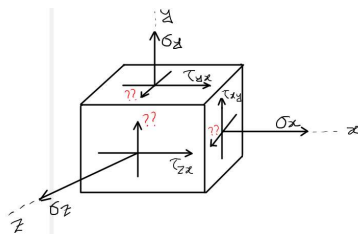


Fig: General Three-Dimensional stress state

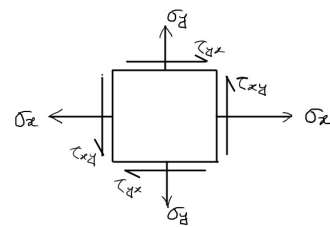


Fig: Plane stress in Equilibrium or cross shear equal (looking from z-axis)

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## Introduction to Stress

## Stress Components

- Total Stress Components, Nine (9)-

$$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}, \tau_{xz}$$

- In most cases, the stresses are considered in **Equilibrium**, cross-shears are equal, i.e.,

$$\tau_{xy} = \tau_{yx}, \quad \tau_{yz} = \tau_{zy}, \quad \tau_{zx} = \tau_{xz}$$

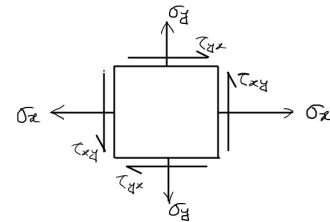
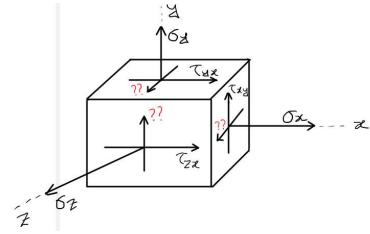
Stress components reduced to six (6) -  $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$

- **Plane stress:** stresses on one surface are equal to zero, i.e.,

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

In equilibrium, plane Stress components reduced to three (3) -

$$\sigma_x, \sigma_y, \tau_{xy}$$



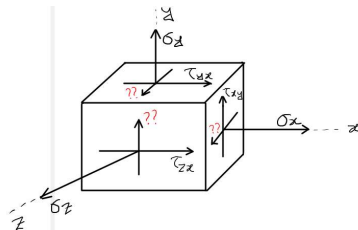
5

## Introduction to Stress

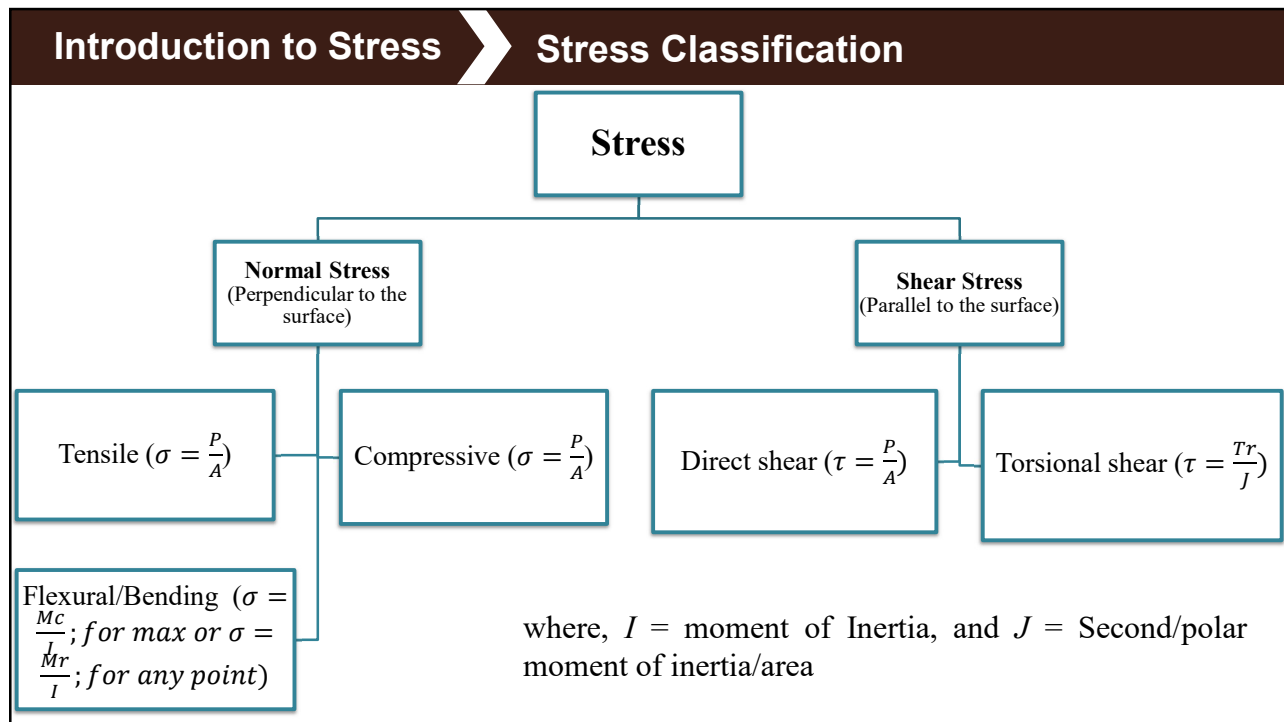
## Stress Components

## QUIZ - 1

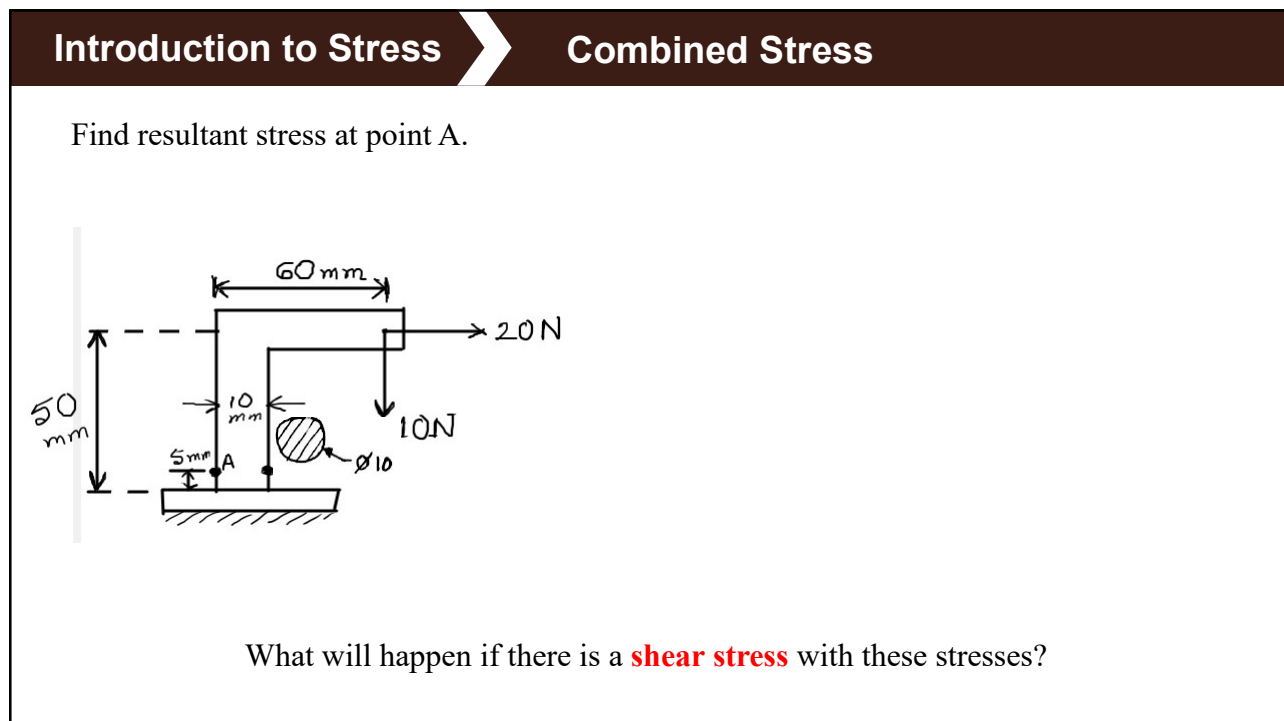
Draw the plane stress in Equilibrium while looking from x-axis and y-axis



6



7



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## Introduction to Stress

## Combined Stress

- Since stress is a **vector quantity**, we can not add or subtract **Normal and Shear** stress arithmetically.
- On a machine component, there can be combined stresses such as-
  - ❖ Tension/compression + Bending (Arithmetically)
  - ❖ Tension/compression + Torsion (Geometrically)
  - ❖ Torsion + Bending (Geometrically)
  - ❖ Tension/compression + Bending + Torsion (Geometrically)

This Geometrical method is nothing but a **Mohr's Circle!**

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## Introduction to Stress

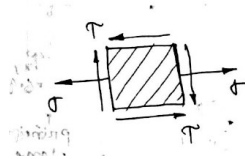
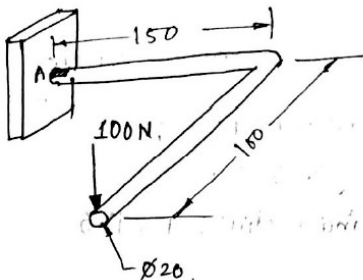
## Combined Stress

## Mohr's Circle

2D Mohr's Circle

- x and y axis represent normal and shear stresses, respectively.
- Angle in circle is twice the actual angle.

Example -



Consider element A,

$$\sigma_x = \frac{Mc}{I} = \frac{100 \times 150 \times 10 \times 64}{\pi(20)^4} = 19.1 \text{ MPa}$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{100 \times 100 \times 10 \times 32}{\pi(20)^4} = 6.4 \text{ MPa}$$

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## Introduction to Stress

## Combined Stress

## Mohr's Circle

Output of a 2D Mohr's circle:➤ **Max. Shear plane and Extreme value shear stress:**

- ❖ Maximum shear plane occurs at  $45^\circ$  to the principal plane. From a 2D Mohr's circle we get two extreme value shear stresses-  $\tau_1, \tau_2$

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- ❖ Point to be noted that, these extreme value shear stresses might not be actual maximum shear stress. Actual maximum shear stress is encouraged to be derived from 'ordered' principal stresses ( $\sigma_1 > \sigma_2 > \sigma_3$ ),  $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$ .
- ❖ However, if the problem becomes **uniaxial**, then it is guaranteed that **Extreme value shear stress = Max. shear stress.**

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## Introduction to Stress

## Combined Stress

## Mohr's Circle

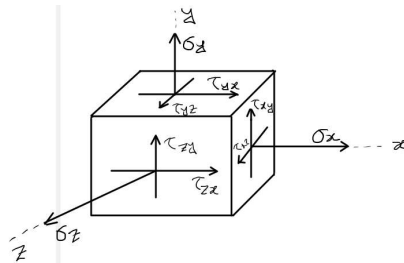
3D Mohr's Circle

Fig: General Three-Dimensional stress state

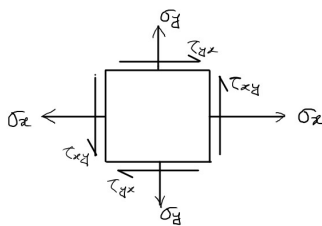


Fig: Plane stress in Equilibrium or cross shear equal (looking from z-axis)

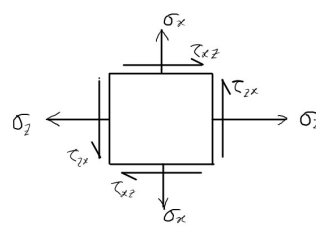


Fig: Plane stress in Equilibrium or cross shear equal (looking from y-axis)

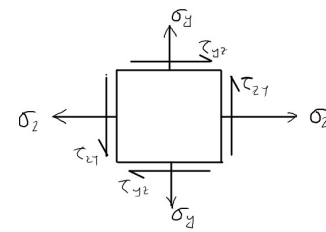


Fig: Plane stress in Equilibrium or cross shear equal (looking from x-axis)

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## Introduction to Stress

## Combined Stress

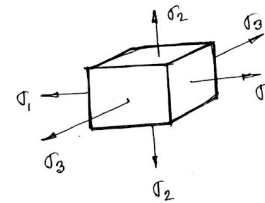
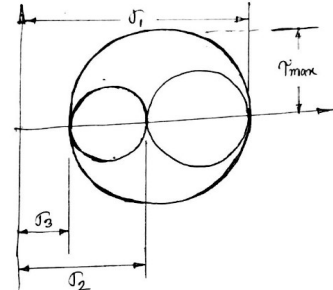
## Mohr's Circle

Output of a 3D Mohr's circle:➤ **Principal stresses and Absolute maximum shear stress:**

- ❖ From the three plane stress state, three 2D Mohr's circle can be drawn. From each 2D Mohr's circle, two principal stresses can be found using the following formula-

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- ❖ From the principal stress values, three unique principal stresses can be found, i.e.,  $\sigma_1, \sigma_2, \sigma_3$  and we can draw a 3D Mohr's circle.
- ❖ Absolute maximum shear stress,  $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$ , where  $\sigma_1 > \sigma_2 > \sigma_3$



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## Introduction to Stress

## Combined Stress

## Mohr's Circle

Practice Problem:

Find  $\tau_{max}$  from the following stresses.

- (a)  $\sigma_x = 70$  kpsi,  $\sigma_y = 70$  kpsi,  $\tau_{xy} = 0$  kpsi
- (b)  $\sigma_x = 60$  kpsi,  $\sigma_y = 40$  kpsi,  $\tau_{xy} = -15$  kpsi
- (c)  $\sigma_x = 0$  kpsi,  $\sigma_y = 40$  kpsi,  $\tau_{xy} = 45$  kpsi
- (d)  $\sigma_x = -40$  kpsi,  $\sigma_y = -60$  kpsi,  $\tau_{xy} = 15$  kpsi
- (e)  $\sigma_1 = 30$  kpsi,  $\sigma_2 = 30$  kpsi,  $\sigma_3 = 30$  kpsi

Hints:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}, \text{ where } \sigma_1 > \sigma_2 > \sigma_3$$

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## Introduction to Stress

## Combined Stress

## Von Mises Stress

Von Mises Stress:

For principal stresses,

$$\sigma' = \left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]^{1/2}$$

For principal stresses in plane,

$$\sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}$$

For general 3D stress state,

$$\sigma' = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}$$

For plane stress in equilibrium,

$$\sigma' = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$

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## Introduction to Stress

## Combined Stress

## Von Mises Stress

**QUIZ – 2**

Find von mises stress,  $\sigma'$  from the following stresses.

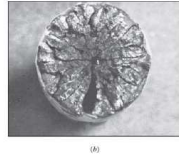
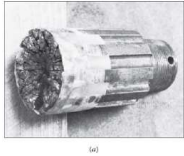
- (a)  $\sigma_x = 70$  kpsi,  $\sigma_y = 70$  kpsi,  $\tau_{xy} = 0$  kpsi
- (b)  $\sigma_x = 60$  kpsi,  $\sigma_y = 40$  kpsi,  $\tau_{xy} = -15$  kpsi
- (c)  $\sigma_x = 0$  kpsi,  $\sigma_y = 40$  kpsi,  $\tau_{xy} = 45$  kpsi
- (d)  $\sigma_x = -40$  kpsi,  $\sigma_y = -60$  kpsi,  $\tau_{xy} = 15$  kpsi
- (e)  $\sigma_1 = 30$  kpsi,  $\sigma_2 = 30$  kpsi,  $\sigma_3 = 30$  kpsi

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## 5-3 Failure Theories

### Failure in Machine Elements:

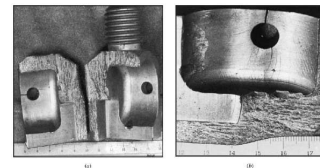
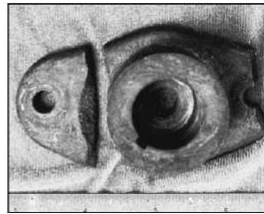
- Failure in machine design means that a part becomes permanently distorted, thus its function is compromised.
- Failure occurs when **stress becomes equal or greater than failure strength**.
  - **Failure strength:** The maximum limit of stress when failure occurs. For example- yield, Ultimate strength, elastic limit, proportional limit, etc.



**Figure 5-1**  
 (a) Failure of a truck drive-shaft spline due to corrosion fatigue. Note that it was necessary to use clear tape to hold the pieces in place. (b) Direct end view of failure. (For permission to reprint Figs. 5-1 through 5-5, the authors are grateful for the personal photographs of Larry D. Mitchell, co-author of *Mechanical Engineering Design*, 4th ed., McGraw-Hill, New York, 1983.)

**Figure 5-2**

Impact failure of a lawnmower blade driver hub. The blade impacted a surveying pipe marker.



**Figure 5-4**  
 Chain link failure that failed in one cycle. To alleviate complaints of excessive wear, the manufacturer decided to case-harden the material. (a) Two links showing fracture; this is an excellent example of brittle fracture initiated by stress concentration. (b) Enlarged view of one portion to show cracks induced by stress concentration at the support-pin holes.

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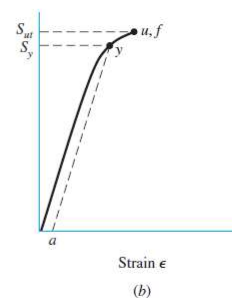
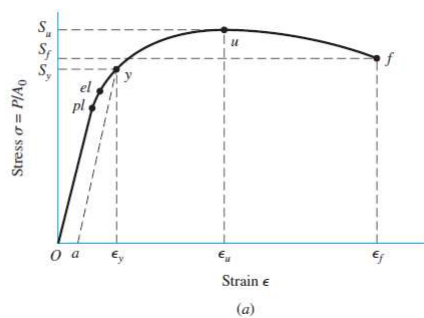
## 5-3 Failure Theories

### Failure in Machine Elements:

- For ductile material, failure strength means yield strength ( $S_{yt}$  or  $S_{yc}$ )
- For brittle material, failure strength means ultimate strength ( $U_t$  or  $U_c$ )

**Figure 2-2**

Stress-strain diagram obtained from the standard tensile test (a) Ductile material; (b) brittle material. *pl* marks the proportional limit; *el*, the elastic limit; *y*, the offset-yield strength as defined by offset strain *a*; *u*, the maximum or ultimate strength; and *f*, the fracture strength.

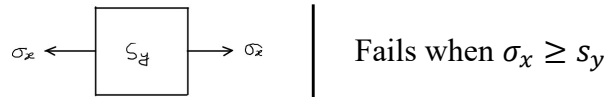


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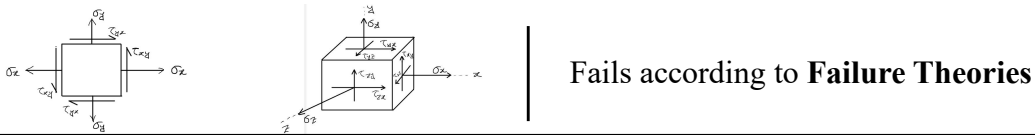
## 5-3 Failure Theories

### Failure in Machine Elements:

- Most of the **ductile materials** are very prone to **maximum shear stress**. That means ductile material fails because of shear stress, at max. shear plane.
- On the contrary, **brittle materials** are very prone to Maximum normal stresses, specifically **principal stresses**. Failure initiates at principal plane.
- For materials subjected to **uniaxial** stress (without shear), it fails when the stress reaches the failure strength (i.e., yield strength, ultimate strength, percent elongation)



- To deal with **biaxial and triaxial** stresses, we need to use **failure theories**.



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## 5-3 Failure Theories

### Some recognized failure theories:

- Failure theories take multi-axial stresses as input and return ‘something’ which can be easily compared to Failure Strengths.
- Because ductile and brittle materials have different failure mechanism, different failure theories are used for each of them.
- **Failure theories for Ductile Material (yield criterion,  $\epsilon_f \geq 0.05$ )**
  - ❖ Ductile materials fail on the maximum shear plane due to maximum shear stress.
  - ❖ Yield strength is considered as the Failure strength of the ductile materials.
  - ❖ Some recognized failure theories for ductile materials-
    - ❑ Max. Shear Stress Theory (MSS)
    - ❑ Distortion Energy Theory (DE)
    - ❑ Ductile Coulomb-Mohr Theory (DCM)

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## 5-3 Failure Theories

### Some recognized failure theories:

#### ➤ Failure theories for Brittle Material (yield criterion, $\epsilon_f < 0.05$ )

- ❖ Brittle materials fail on the principal plane due to principal stress.
- ❖ Ultimate strength is considered as the Failure strength of the brittle materials.
- ❖ Some recognized failure theories for ductile materials-

- Max. Normal Stress Theory (MNS)
- Brittle Coulomb-Mohr Theory (BCM)
- Modified-Mohr Theory (MM)

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## 5-4 Maximum-Shear-Stress Theory for Ductile Materials

### MSS Theory:

- According to MSS, failure occurs when –

$$\tau_{max} \geq S_{sy} \text{ and } S_{sy} = 0.5S_y$$

- Design equation-

$$\tau_{max} = \frac{0.5S_y}{n}$$

- More conservative than other failure theories.

Where,

$\tau_{max}$  = Maximum shear stress

$S_y$  = Yield strength

$S_{sy}$  = Yield strength in shear

$n$  = Factor of safety

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## 5-5 Distortion-Energy Theory for Ductile Materials

### DE Theory:

- According to DE, failure occurs when –

$$\sigma' \geq S_y$$

- Design equation-

$$\sigma' = \frac{S_y}{n}$$

- According to DE,  $S_{sy} = 0.577S_y$

Where,

$\sigma' = \text{Von Mises stress}$

$S_y = \text{Yield strength}$

$S_{sy} = \text{Yield strength in shear}$

$n = \text{Factor of safety}$

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## 5-5 Distortion-Energy Theory for Ductile Materials

**EXAMPLE 5-1** A hot-rolled steel has a yield strength of  $S_y = S_{yc} = 100$  kpsi and a true strain at fracture of  $\epsilon_f = 0.55$ . Estimate the factor of safety for the following principal stress states:

(a)  $\sigma_x = 70$  kpsi,  $\sigma_y = 70$  kpsi,  $\tau_{xy} = 0$  kpsi  
 (b)  $\sigma_x = 60$  kpsi,  $\sigma_y = 40$  kpsi,  $\tau_{xy} = -15$  kpsi  
 (c)  $\sigma_x = 0$  kpsi,  $\sigma_y = 40$  kpsi,  $\tau_{xy} = 45$  kpsi  
 (d)  $\sigma_x = -40$  kpsi,  $\sigma_y = -60$  kpsi,  $\tau_{xy} = 15$  kpsi  
 (e)  $\sigma_1 = 30$  kpsi,  $\sigma_2 = 30$  kpsi,  $\sigma_3 = 30$  kpsi

### Hints

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}, \text{ where } \sigma_1 > \sigma_2 > \sigma_3$$

$$\sigma' = \left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]^{1/2}$$

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$

$$\sigma' = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}$$

$$\sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$

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## 5-6 Coulomb-Mohr Theory for Ductile Materials

### DCM Theory:

- According to DCM, when  $S_{yt} \neq S_{yc}$  for any material, failure condition is–

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \geq 1$$

- Design equation–

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} = \frac{1}{n}$$

- According to DCM,

$$S_{sy} = \frac{S_{yt} S_{yc}}{S_{yt} + S_{yc}}$$

Where,

$\sigma_1, \sigma_3$  are Principal stresses

$S_{yt}$  = Tensile yield strength

$S_{yc}$  = Compression yield strength

$S_{sy}$  = Yield strength in shear

$n$  = Factor of safety

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## Failure Theory for Ductile Materials

### QUIZ – 3

#### EXAMPLE 5-2

A 40-mm-diameter shaft is statistically torqued to 350 Nm. It is made of cast 195-T6 aluminum with a yield strength in tension of 160 MPa and a yield strength in compression of 170 MPa. Estimate the Factor of safety of the shaft using –

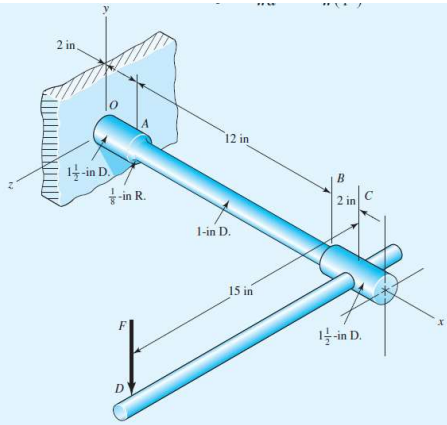
- i. DCM Theory
- ii. MSS Theory

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## 5-5 Distortion-Energy Theory for Ductile Materials

### EXAMPLE 5-3

A certain force  $F$  applied at  $D$  near the end of the 15-in lever shown in Fig. 5-16, which is quite similar to a socket wrench, results in certain stresses in the cantilevered bar  $OABC$ . This bar ( $OABC$ ) is of AISI 1035 steel, forged and heat-treated so that it has a minimum (ASTM) yield strength of 81 kpsi. We presume that this component would be of no value after yielding. Thus the force  $F$  required to initiate yielding can be regarded as the strength of the component part. Find this force.

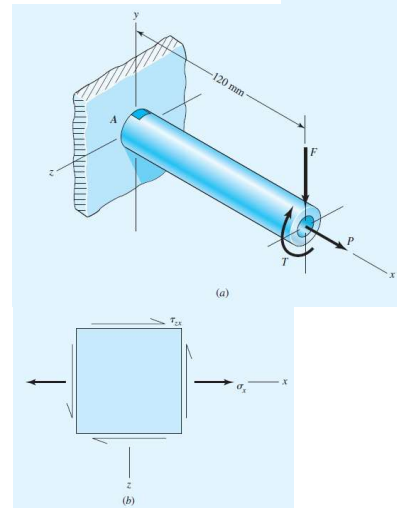


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## 5-5 Distortion-Energy Theory for Ductile Materials

### EXAMPLE 5-4

The cantilevered tube shown in Fig. 5-17 is to be made of 2014 aluminum alloy treated to obtain a specified minimum yield strength of 276 MPa. We wish to select a stock-size tube from Table A-8 using a design factor  $n_d = 4$ . The bending load is  $F = 1.75$  kN, the axial tension is  $P = 9.0$  kN, and the torsion is  $T = 72$  N · m. What is the realized factor of safety?



**Practice Yourself!**

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## 5-8 Maximum-Normal-Stress Theory for Brittle Materials

### MNS Theory:

- According to MNS, failure occurs whenever one of the three principal stresses becomes equal or exceeds the Ultimate tensile or compressive strength.

$$\sigma_1 \geq S_{ut} \quad \text{or} \quad \sigma_3 \leq -S_{uc}$$

- Design equation-

$$\sigma_1 = \frac{S_{ut}}{n}$$

$$\sigma_3 = \frac{-S_{uc}}{n}$$

Where,

$\sigma_1, \sigma_3$  are Principal stresses

$S_{ut}$  = Ultimate tensile strength

$S_{uc}$  = Ultimate compressive strength

$n$  = Factor of safety

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## 5-9 Modifications of the Mohr Theory for Brittle Materials

### Brittle-Coulomb-Mohr (BCM) Theory:

- According to BCM, failure condition is–

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} \geq 1$$

- Design equation-

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n}$$

Where,

$\sigma_1, \sigma_3$  are Principal stresses

$S_{ut}$  = Ultimate tensile strength

$S_{uc}$  = Ultimate compressive strength

$n$  = Factor of safety

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## 5-9 Modifications of the Mohr Theory for Brittle Materials

### Modified Mohr (MM) Theory:

➤ According to MM theory, design equations are—

$$\sigma_A = \frac{S_{ut}}{n} \quad \text{when, } \begin{cases} \sigma_A \geq \sigma_B \geq 0 \\ \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left| \frac{\sigma_B}{\sigma_A} \right| \leq 1 \end{cases}$$

$$\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \quad \text{when, } \begin{cases} \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left| \frac{\sigma_B}{\sigma_A} \right| > 1 \end{cases}$$

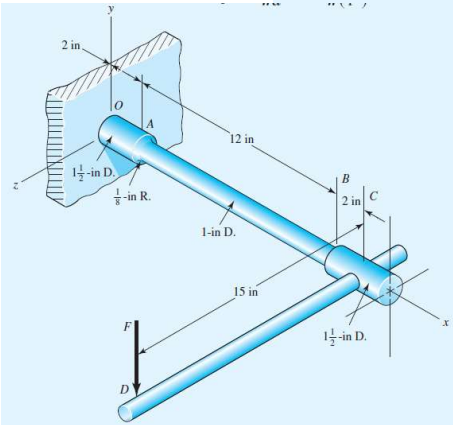
$$\sigma_B = -\frac{S_{uc}}{n} \quad \text{when, } \begin{cases} 0 \geq \sigma_A \geq \sigma_B \end{cases}$$

Where,  
 $\sigma_A, \sigma_B$  are two non-zero principal stresses for plane stress  
 $S_{ut}$  = Ultimate tensile strength  
 $S_{uc}$  = Ultimate compressive strength  
 $n$  = Factor of safety

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## 5-9 Modifications of the Mohr Theory for Brittle Materials

### EXAMPLE 5-5



Consider the wrench in Ex. 5-3, Fig. 5-16, as made of cast iron, machined to dimension. The force  $F$  required to fracture this part can be regarded as the strength of the component part. If the material is ASTM grade 30 cast iron, find the force  $F$  with  
 (a) Coulomb-Mohr failure model.  
 (b) Modified Mohr failure model.

Table A-24

Mechanical Properties of Three Non-Steel Metals  
 (a) Typical Properties of Gray Cast Iron

[The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 20 kpsi. Note particularly that the tabulations are *typical* of several heats.]

ASTM Number	Tensile Strength $S_{ut}$ , kpsi	Compressive Strength $S_{uc}$ , kpsi	Shear Modulus of Rupture $S_{ur}$ , kpsi	Modulus of Elasticity, Mpsi		Endurance Limit* $S_e$ , kpsi	Brinell Hardness $H_b$	Fatigue Stress-Concentration Factor $K_f$
				Tension <sup>†</sup>	Torsion			
20	22	83	26	9.6-14	3.9-5.6	10	156	1.00
25	26	97	32	11.5-14.8	4.6-6.0	11.5	174	1.05
30	31	109	40	13-16.4	5.2-6.6	14	201	1.10
35	36.5	124	48.5	14.5-17.2	5.8-6.9	16	212	1.15
40	42.5	140	57	16-20	6.4-7.8	18.5	235	1.25
50	52.5	164	73	18.8-22.8	7.2-8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4-23.5	7.8-8.5	24.5	302	1.50

\*Polished or machined specimens.

<sup>†</sup>The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

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## 5-11 Selection of Failure Criteria

**Figure 5-21**

Failure theory selection flowchart.

